F THE MODULUS FUNCTION

MODULUS

The **modulus** or **absolute value** of a real number is its size, ignoring its sign.
We denote the modulus of x by $|x|$.

For example, the modulus of 7 is 7, and the modulus of -7 is also 7, so we write $|7| = 7$ and $|-7| = 7$.

Example 9

Self Tutor

If $a = -7$ and $b = 3$, find:

a $|a + b|$

b $|ab|$

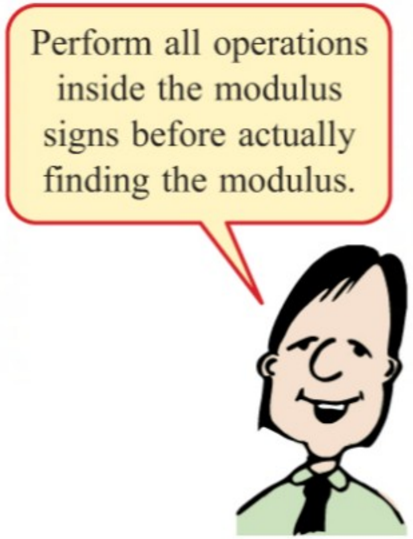
Example 9 **Self Tutor**

If $a = -7$ and $b = 3$, find:

a $|a + b|$ **b** $|ab|$

a
$$\begin{aligned} &|a+b| \\ =\ &|-7+3| \\ =\ &|-4| \\ =\ &4 \end{aligned}$$

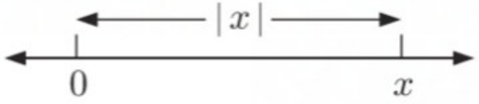
b
$$\begin{aligned} &|ab| \\ =\ &|-7 \times 3| \\ =\ &|-21| \\ =\ &21 \end{aligned}$$


Perform all operations inside the modulus signs before actually finding the modulus.

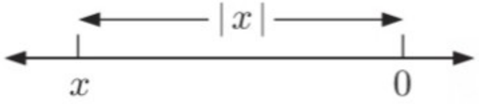
GEOMETRIC DEFINITION OF MODULUS

$|x|$ is the distance of x from 0 on the number line.
Because the modulus is a distance, it cannot be negative.

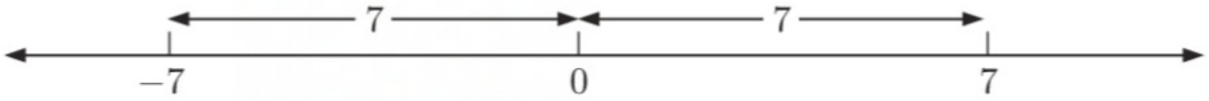
If $x > 0$:



If $x < 0$:



For example:

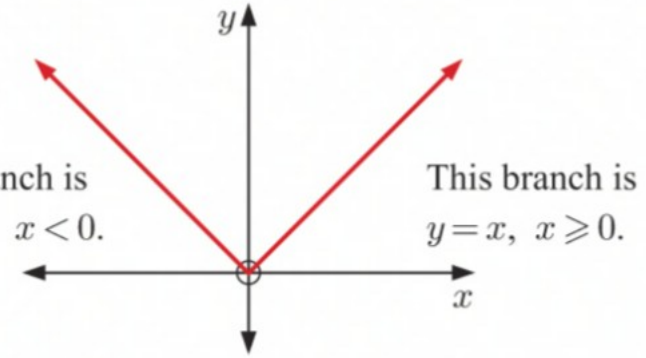


$|-7| = |7| = 7$

ALGEBRAIC DEFINITION OF MODULUS

From the **Investigation**, you should have found that: $|\boldsymbol{x}| = \begin{cases} \boldsymbol{x} & \text{if } \boldsymbol{x} \geqslant \boldsymbol{0} \\ -\boldsymbol{x} & \text{if } \boldsymbol{x} < \boldsymbol{0} \end{cases}$ or $|\boldsymbol{x}| = \sqrt{\boldsymbol{x}^2}$

$y = |x|$ has graph:



To draw graphs involving $|x|$, we must consider the cases $x \geqslant 0$ and $x < 0$ separately, so that we can write the function without the modulus sign.

Example 10

Self Tutor

Draw the graph of $f(x) = x + 2\,|x|$.

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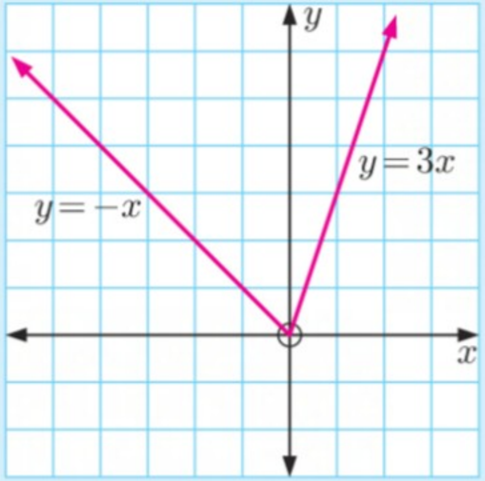
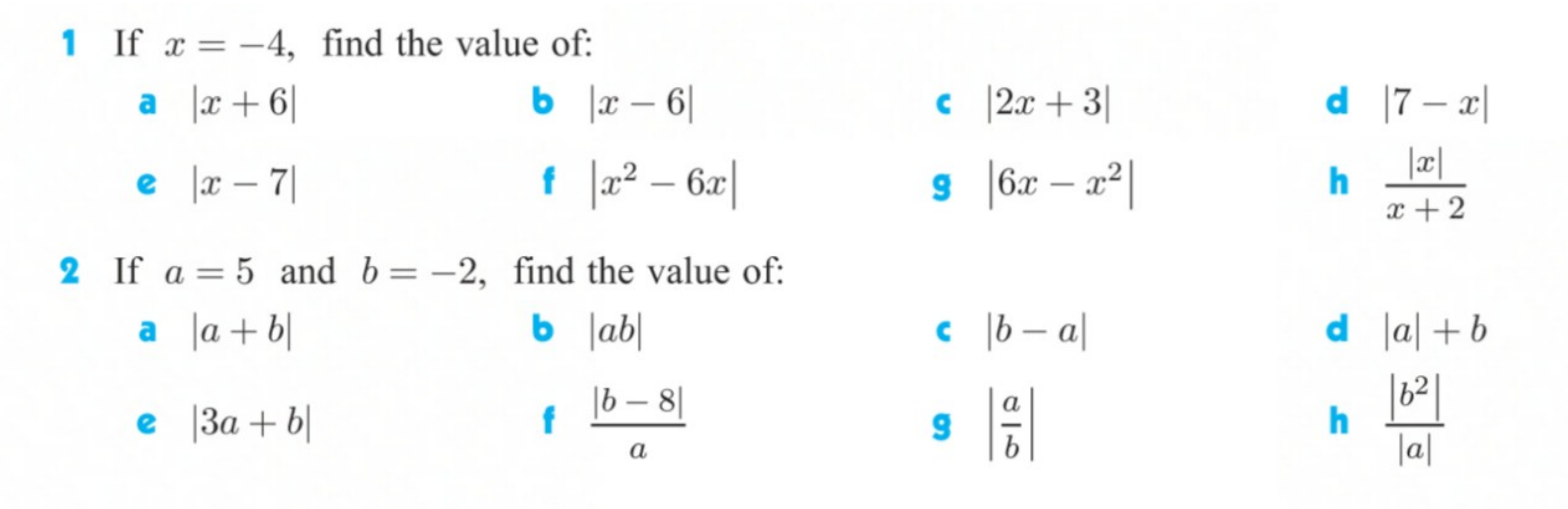
Self Tutor

Draw the graph of $f(x) = x + 2\,|x|$.

$$\begin{aligned} f(x) &= x + 2\,|x| \\ &= \begin{cases} x + 2(x) & \text{if } x \geqslant 0 \\ x + 2(-x) & \text{if } x < 0 \end{cases} \\ &= \begin{cases} 3x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases} \end{aligned}$$





1 If $x = -4$, find the value of:

- **a** $|x + 6|$
- **b** $|x - 6|$
- **c** $|2x + 3|$
- **d** $|7 - x|$
- **e** $|x - 7|$
- **f** $\left|x^2 - 6x\right|$
- **g** $\left|6x - x^2\right|$
- **h** $\dfrac{|x|}{x + 2}$

2 If $a = 5$ and $b = -2$, find the value of:

- **a** $|a + b|$
- **b** $|ab|$
- **c** $|b - a|$
- **d** $|a| + b$
- **e** $|3a + b|$
- **f** $\dfrac{|b - 8|}{a}$
- **g** $\left|\dfrac{a}{b}\right|$
- **h** $\dfrac{\left|b^2\right|}{|a|}$

3 **a** Copy and complete:

x	9	3	0	-3	-9
x^2					
$\|x\|^2$					

b What can you conclude from **a**?

4 By replacing $|x|$ with x for $x \geqslant 0$ and $(-x)$ for $x < 0$, write the following functions without the modulus sign. Hence, graph each function:

a $f(x) = -|x|$

b $f(x) = |x| + x$

c $f(x) = |x| + 2$

d $f(x) = x - 2|x|$

e $f(x) = 3|x| + 1$

f $f(x) = 5 - |x|$

g $f(x) = |x|^2 - 4$

h $f(x) = \dfrac{|x|}{x}$

i $f(x) = \sqrt{|x|}$

5 **a** Use technology to graph:

i $y = |(x-2)(x-4)|$

ii $y = |x(x-3)|$

b Explain how these functions are related to $y = (x-2)(x-4)$ and $y = x(x-3)$.

GRAPHING PACKAGE

1 **a** 2 **b** 10 **c** 5 **d** 11 **e** 11 **f** 40
g 40 **h** -2

2 **a** 3 **b** 10 **c** 7 **d** 3 **e** 13 **f** 2
g $\frac{5}{2}$ **h** $\frac{4}{5}$

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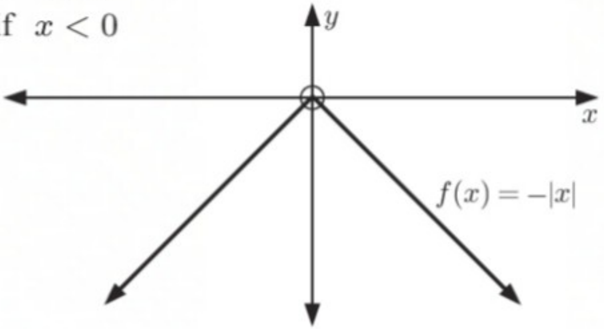
3 **a**

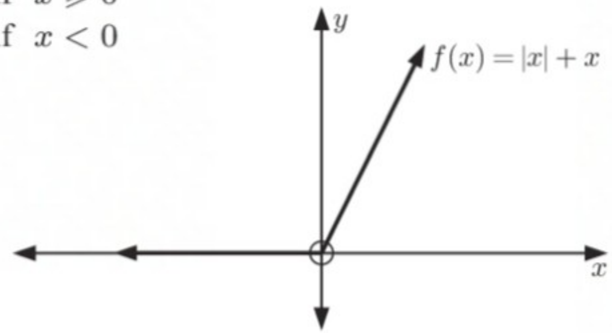
x	9	3	0	-3	-9
x^2	81	9	0	9	81
$\lvert x\rvert^2$	81	9	0	9	81

b $x^2 = \lvert x\rvert^2$

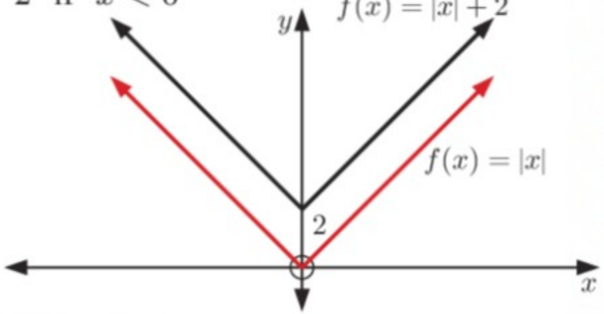
4 **a** $f(x) = \begin{cases} -x & \text{if } x \geqslant 0 \\ x & \text{if } x < 0 \end{cases}$



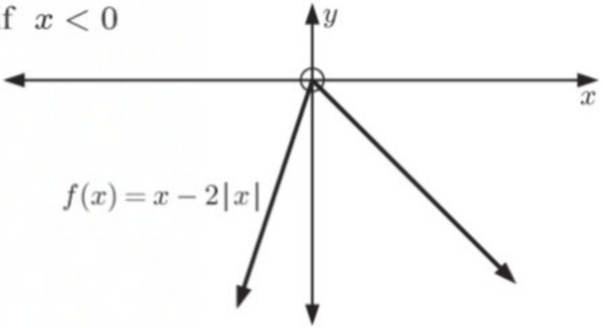
b $f(x) = \begin{cases} 2x & \text{if } x \geqslant 0 \\ 0 & \text{if } x < 0 \end{cases}$



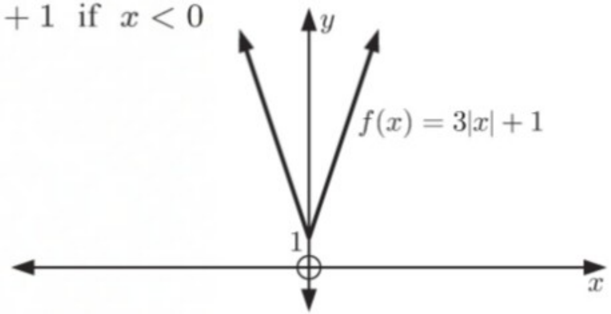
c $f(x) = \begin{cases} x + 2 & \text{if } x \geqslant 0 \\ -x + 2 & \text{if } x < 0 \end{cases}$



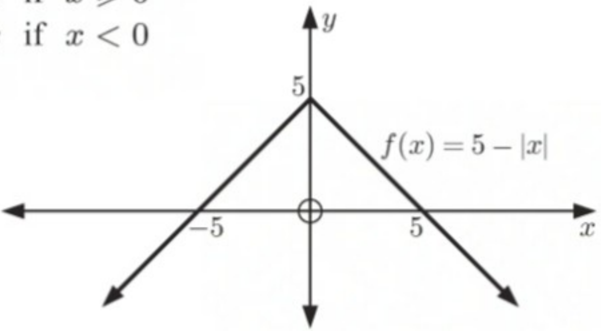
d $f(x) = \begin{cases} -x & \text{if } x \geqslant 0 \\ 3x & \text{if } x < 0 \end{cases}$



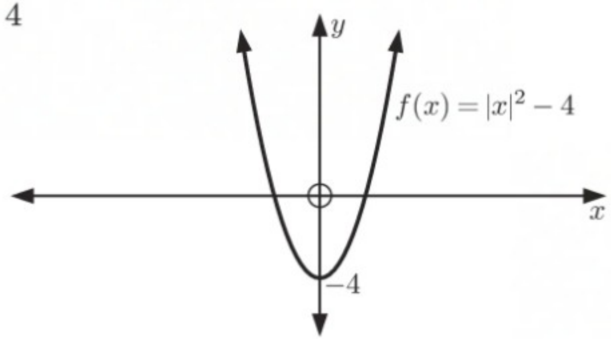
e $f(x) = \begin{cases} 3x + 1 & \text{if } x \geqslant 0 \\ -3x + 1 & \text{if } x < 0 \end{cases}$



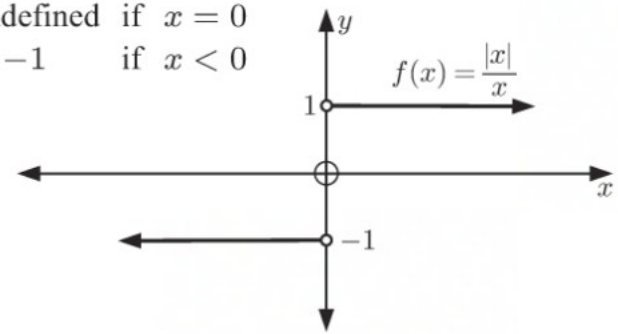
f $f(x) = \begin{cases} 5 - x & \text{if } x \geqslant 0 \\ 5 + x & \text{if } x < 0 \end{cases}$



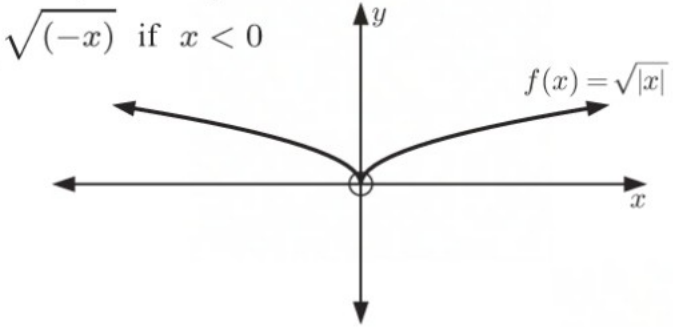
g $f(x) = x^2 - 4$



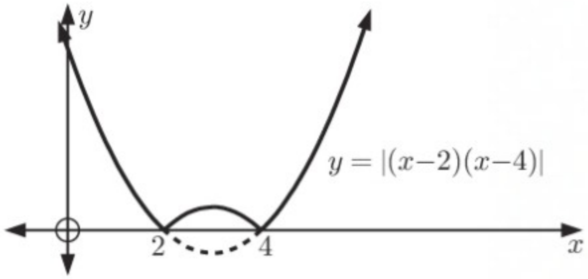
h $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ \text{undefined} & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$



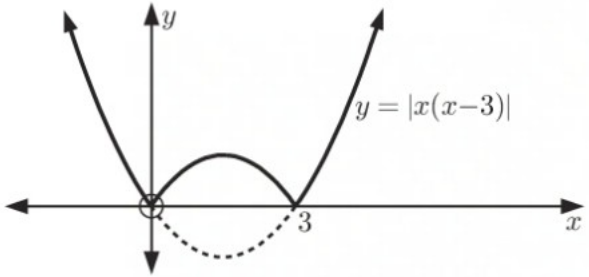
i $f(x) = \begin{cases} \sqrt{x} & \text{if } x \geqslant 0 \\ \sqrt{(-x)} & \text{if } x < 0 \end{cases}$



5 **a** **i**



ii



b The part of the graph $y = (x-2)(x-4)$ or $y = x(x-3)$ below the x-axis is reflected in the x-axis.